quencies. For no airflow, these frequencies are given by

$$\Omega^2 = 2[\bar{\beta}^2 - \bar{p}(1 + r\bar{\beta})]/[1 + (r + \bar{J})\bar{\beta} - r\bar{J}\bar{p} \pm ([1 + (r - \bar{J})\bar{\beta} + r\bar{J}\bar{p}]^2 + 4\bar{J}\bar{\beta})^{1/2}] \quad (7)$$

where

$$\bar{\beta} = (m/\theta)^2 + n^2 \qquad \bar{p} = (m/\theta)^2 k_x + n^2 \bar{N}_y$$

The frequencies given by the smaller solution for Ω^2 are referred to as the bending set of frequencies because for $r = \bar{J} = 0$ (shear flexibility and rotary inertia neglected) they reduce to the frequencies of the pure bending motion given by classical plate theory; the large solution for Ω^2 gives the frequencies of the thickness-shear set of modes. (Both sets are described in Ref. 6.) In the case of a beam, the bending and thickness-shear sets of frequencies are both predicted by Timoshenko's beam theory[7] and have been discussed in Ref. 8.

Some numerical results obtained from the solutions given by Eq. (5a) and the quartic part of Eq. (1) are shown by the solid curves in Figs. 1 and 2 for $\bar{J}$ equal to 0.05 and 0.50, respectively, and with $r = 1$, $k_x = -4$, $\bar{N}_y = 0$, $n = \theta = 1$ in both cases. (Bm and TSm denote the mth bending and thickness-shear frequencies, respectively.) For comparison, results presented in Fig. 4 of Ref. 1 are shown by the dashed curves. The analysis of Ref. 1 predicts that the first two bending frequencies coalesce whereas the present analysis shows that $B1$ and $B2$ are uncoupled for the parameter values selected. This change in frequency coalescence behavior is due to the presence of the TS frequencies and is discussed in greater detail in Ref. 5. (The thickness-shear and the thickness-twist frequencies are not predicted by the analysis of Ref. 1.)

An additional effect of the rotary inertia is that it makes the solution for λ_{cr} dependent on $\bar{N}_y$. This differs from the solution of Ref. 3 ($\bar{J} = 0$) where the frequency and the crossflow loading term always grouped in the characteristic equation as $\Omega^2 + n^2\bar{N}_y$, a change in $\bar{N}_y$ merely shifting the frequency loops along the Ω^2 axis. This unique grouping does not occur for $\bar{J} > 0$ [see Eq. (1)]. Numerical results in Ref. 5 indicate, however, that $\bar{N}_y$ has a relatively small effect on frequency coalescence.

It should also be noted that values for $\bar{J} > 0.01$ are probably not representative of typical sandwich construction.† For panels having face sheets that are thin compared to the core thickness (an implicit assumption in the panel theory of Ref. 4 since the face sheet bending stiffness is neglected), $\bar{J}$ is given, approximately, by

$$\bar{J} \approx (\pi/2)^2(h/b)^2[1 + \tfrac{1}{2}(\rho_c h/\rho_s t_s)]/[1 + (\rho_c h/\rho_s t_s)]$$

where h is the core thickness and $(\rho_c h)/(\rho_s t_s)$ is the ratio of the core weight to the weight of the two face sheets (per unit area). Thus, $\bar{J} = 0.5$ (Fig. 2) requires the core thickness to be of the same order of magnitude as the panel width b.

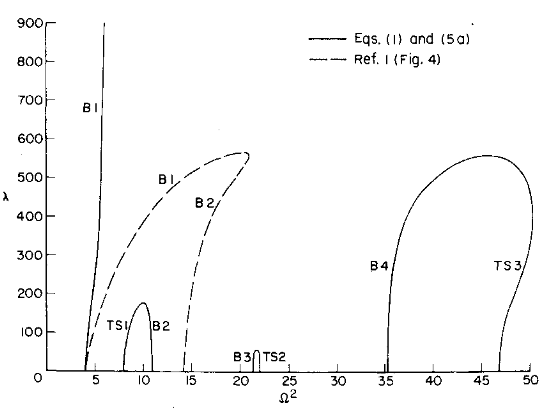

Fig. 2 Frequency loops for $\bar{J} = 0.5$; $r = 1$, $k_x = -4$, $\bar{N}_y = 0$, $n = \theta = 1$.

† Results are presented in Ref. 5 for the range $0 \leq \bar{J} \leq 0.01$. (In Ref. 5, $\bar{J}$ is called χ.)

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[1] Marafioti, F. A. and Johnston, E. R. Jr., "Effects of Rotary Inertia on the Supersonic Flutter of Sandwich Panels," *AIAA Journal*, Vol. 9, No. 2, Feb. 1971, pp. 245–249.

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Reply by Authors to L. L. Erickson

FRANK A. MARAFIOTI* AND E. RUSSELL JOHNSTON JR.†
University of Connecticut, Storrs, Conn.

ERICKSON claims that an inconsistency exists in Ref. 1 because the effect of shear on rotary inertia was omitted. By including the terms $[J/D(Q_x/D_Q)]_{,tt}$ and $[J/D(Q_y/D_Q)]_{,tt}$, Erickson has extended the work of Ref. 1 to include the effect of shear on rotary inertia. It was pointed out in Ref. 1 that the effect of transverse shear on the rotary inertia term was neglected (p. 246, second column, Ref. 1). The authors feel that the rotary inertia moments are presented in a consistent manner. Equation (C3) of Ref. 2 discusses the boundary conditions that are to be applied to a typical sandwich panel and does not imply that transverse shear must be considered for rotary inertia effects. The stress resultants and couples used in Ref. 1 consider the effect of transverse shear; therefore, the correct panel deformation patterns are obtained and shear behavior is considered.

In Ref. 3, Erickson and Anderson did not include the effects of rotary inertia in solving the flutter problem. In Ref. 1, Marafioti and Johnston include the effects of the rotary inertia terms $(J/D)w_{,xtt}$ and $(J/D)w_{,ytt}$. The results obtained in Ref. 1 show that in all cases the effect of these additional terms is to lower the critical dynamic pressure by 10 to 20% (Figs. 5 and 6, p 249, Ref. 1). Erickson now includes an addition term in the rotary inertia expressions, which become

$$J/D[w_{,x} - (Q_x/D_Q)]_{,tt} \text{ and } J/D[w_{,y} - (Q_y/D_Q)]_{,tt}$$

His results show differences of as much as 70% when compared to Ref. 1, and even larger differences from the results presented in Ref. 3. In Ref. 3, the dynamic pressure was calculated to be approximately 1050 (Fig. 5, Ref. 1); whereas,

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* NASA fellow, Civil Engineering Department.
† Professor of Civil Engineering.

now the dynamic pressure has been calculated as low as 190. It would appear that the large differences reported by Erickson are unexpected, since both parts of the rotary inertia terms represent low-order effects.

Since one of the reasons for including rotary inertia was to attempt to account for the anomalous behavior of the curves of critical dynamic pressure vs shear flexibility for $J = 0$ (see Figs. 5 and 6, Ref. 1), it would be interesting to see Erickson's results for these cases. The authors wish to thank Erickson for correcting the typographical errors in Eq. (23) of Ref. 1.

References

[1] Marafioti, F. A. and Johnston, E. R., Jr., "Effects of Rotary Inertia on the Supersonic Flutter of Sandwich Panels," *AIAA Journal*, Vol. 9, No. 2, Feb. 1971, pp. 245–249.

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[3] Erickson, L. L. and Anderson, M. S., "Supersonic Flutter of Simply Supported Isotropic Sandwich Panels," TN D-3171, April 1966, NASA.

Errata: "Exhaust Characteristics of a Megawatt Nitrogen MPD-ARC Thruster"

Charles J. Michels and Donald R. Sigman
NASA Lewis Research Center, Cleveland, Ohio

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In Figs. 7 and 8, and in the text on pp. 1146 and 1147, wherever peak arc-current is shown to be equal to 7.4 kA or 13.4 kA, the values should be corrected to read 11.2 kA and 20.0 kA, respectively. Also, in the second line on p. 1145, the words "naut miles" should read nanometers.

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